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DISCUSSION OF PROCEEDINGS - SEPARATES

482, 499, 500, 596, 597, 598

HIGHWAY DIVISION

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Discussion of
"EQUATION OF THE FREE-FALLING NAPPE"

by Fred W. Blaisdell
(Proc. Sep. 482)

FRED W. BLAISDELL,[1] M. ASCE.—Professor Rand is, of course, correct when he states that the equations presented in the paper apply only while the nappe is free-falling. They do not apply while the nappe is supported by the water cushion in back of the nappe or after the nappe plunges into a pool. Although gravity forces may accelerate the nappe when the nappe is supported by the water cushion in back of it, gravity can have no effect on the nappe shape after it plunges into a tailwater pool. The writer has developed an equation[2] for the form of the upper nappe trajectory below the tailwater level by assuming that the nappe undergoes no further vertical acceleration after it passes through the surface of the tailwater. In other words, the slope of the nappe below the tailwater surface is the same as its slope when it enters the tailwater. This discussion is an extension of Professor Rand's remarks to a more severe and probably more usual case.

The distance from the weir at which the tangent to the non-submerged upper nappe will become parallel to the apron is useful as a reference point for the location of energy-dissipating baffles, as stated by Professor Rand. Mr. Donnelly and the writer have found that the average distance at which the free and submerged nappes strike the apron is also a good reference point for the location of baffles and sills.[2] The actual point to use will probably depend on whether there is no submergence of the nappe, as in Professor Rand's case, or whether the nappe is submerged, as in the case of the straight drop spillway stilling basin.

The information presented by Mr. Harrold is most interesting and is a valuable addition to the paper. The agreement between the author's equation, which was developed from data obtained from numerous sources, and the Corps of Engineers equation, developed from the Bureau of Reclamation data alone, is shown by Mr. Harrold to be excellent. This should engender confidence in both equations.

The writer wishes to thank Professor Rand and Mr. Harrold for taking time to make their interesting and constructive comments.

1. Project Supervisor, U. S. Dept. of Agriculture, St. Anthony Falls Hydr. Lab., Minneapolis, Minn.
2. Straight Drop Spillway Stilling Basin, by Charles A. Donnelly and Fred W. Blaisdell, University of Minnesota, St. Anthony Falls Hydraulic Laboratory, Technical Paper No. 15-B, November, 1954.

Discussion of
"PERFORMANCE OF SOILS VERSUS CHEMICAL PROPERTIES"

by C. Kinney Hancock
(Proc. Sep. 499)

EUGENE Y. HUANG,[1] A.M. ASCE.—Based on data obtained from laboratory tests and field observations, Professor Hancock has found a relationship between the silica-sesquioxide ratio of a soil and the performance of the soil when it was used as a roadbed material. Consequently the soil colloid silica-sesquioxide ratio has been proposed as a supplementary index to roadbed performance.

Inasmuch as the silica-sesquioxide ratio is determined for colloidal clay which is known to be the most chemically active fraction of a soil, there is no doubt but what this ratio is to a certain extent an index of some practical importance. In addition to the references enumerated by Professor Hancock, this relationship is revealed by the findings of Professor Winterkorn which were disclosed in 1935 and 1936.[2] In his studies on the surface-chemical factors influencing the engineering properties of soils, Professor Winterkorn has shown that the silica-sesquioxide ratio of several soils has a definite relationship with their physical constants, tensile and compressive strengths, and other physical characteristics and properties. It appears, however, that the presently available data regarding this relationship is far from sufficient to develop a general conclusion applicable to the engineering use of soils. Much more research along this line certainly would be necessary to the establishment of a useful correlation. In this connection, the writer believes that the colloidal behavior of a soil is probably dependent largely upon a few basic factors such as clay minerals, exchangeable bases, organic matter, and the like, to which the silica-sesquioxide ratio is more or less related, and that the silica-sesquioxide ratio or the double percentage ratio ($2\%SiO_2/\%R_2O_3$), as proposed by Professor Hancock, is probably not in itself alone a complete criterion and therefore can only be considered in relation to a number of other factors for dealing with roadbed soil problems.

In regard to the performance of a roadbed, which is dependent upon a combination of circumstances, the writer cannot accept the idea that either the silica-sesquioxide ratio as shown in Figs 1 to 3 or the physical constants (including Atterberg limits and shrinkage factors) of the roadbed soil alone may be used for its evaluation. The basic requirements for satisfactory roadbed soil materials are (1) sufficient mechanical stability to resist displacement under load under expected weather conditions and (2) ability to

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2. "Adsorption Phenomena in Relation to Soil Stabilization," by Hans F. Winterkorn, Proceedings of Highway Research Board, Vol. 15, 1935, pp. 343-355.
"Surface-Chemical Factors Influencing The Engineering Properties of Soils," by Hans F. Winterkorn, Proceedings Highway Research Board, Vol. 16, 1936, pp. 293-308.

withstand a certain amount of volume change due to fluctuations in moisture content and temperature. An unsatisfactory roadbed may result from undesirable soil material, inadequate compaction, a high water table, an insufficient drainage system, frost heaving and frost boils, or a combination of such things. Failures may also occur, however, from an inadequate road structure, such as insufficient thickness or poor quality of surface course. In most cases, a combination of several factors with differing degrees of importance is what causes a roadbed to develop defects. The precise cause of a "good" or a "poor" roadbed in a given instance can, therefore, be determined only after a thorough observation of site conditions and a careful examination of construction records, plus, if necessary, a detailed laboratory study of the soil material. It seems very unlikely that roadbed performance can be directly correlated with the silica-sesquioxide ratio, a value which is associated only with the colloidal fraction of the material in the roadbed and which is in no way a measure of the many other important variables known to affect performance.

In typical highway construction the roadbed material is composed of a mixture of soil and aggregates, the texture of which ranges from colloidal clay to coarse gravel. From the standpoint of stability, as far as the roadbed is concerned, primary consideration should be given to the source of resistance, both cohesion and internal friction, in the soil mixture. The granular fraction is assumed to provide an internal skeleton of grains of sand and larger particles touching each other or interlocking with each other to furnish internal friction while the soil fines act as a filler and cementing medium to bind all the fractions into a mass. The load-carrying ability of a soil mixture depends upon the strength and nature of its individual particles and probably even more on the mechanical properties of the system or arrangement of the particles. Since the Atterberg limits and shrinkage factors indicate a soil's capacity to develop cohesion and the degree of volume change with varying moisture content, they are often used as qualitative tests for the soil fraction passing the No. 40 sieve in selecting the types of soil which are suitable to be used as roadbed material as well as for determining the amount of binder soil which is necessary to supply adequate cohesion without overfilling the voids between the granular particles of the mixture. The structural strength and stability of a roadbed, under most conditions, are dependent upon both its cohesion and internal friction. A criterion for roadbed performance evaluation or prediction can hardly be considered entirely reliable if it is based solely upon the physical constants of the finer fractions. It may be interesting to note from Table I of Professor Hancock's paper that the amount of material retained on the No. 40 sieve ranged from 26 to 57 per cent, indicating the granular character of the mixtures, the effect of which, according to the writer's opinion, should not be overlooked in evaluating roadbed materials.

Discussion of
"EXPEDIENT GEOMETRY FOR THE DESIGN OF
COMPOUND HIGHWAY CURVES WITH CONNECTION SPIRALS"

by Robert D. Schacherl and George J. Berindoague
(Proc. Sep. 500)

PAUL HARTMAN,[1] M. ASCE.—The design procedure is excellent but it seems unnecessarily complicated by the rigid adherence to the selected tangents. The location of these tangents is not so precise as to prohibit small parallel shifts. Such shifts make possible the use of curves of even degree and simplify both office and field computations.

A modification of the procedure which is outlined below, permits complete determination of the characteristics of the curves and spirals, using slide rule computation only. Equal-length spirals are used except where it is desirable to make a curve transitional throughout. The use of unequal-length spirals introduces no complications as will be shown.

The proposed procedure is briefly this: 1) ignoring spirals, compute the degree of each curve in turn by dividing the desired tangent length into the tangent length of a 1^{o} curve with the same central angle; 2) round off the computed value to the nearest whole degree and compute the tangent lengths by dividing the 1^{o} curve tangent by the degree of curve; 3) compare the resulting distances between P.I.'s with the given distances and change the degree of curve (using whole degrees only) where necessary to make distances roughly equal. The computed distances should in general be a few feet smaller than the given distances but an exception will not require revision of degree of curve, particularly where the intersection angles are small; 4) compute the lengths of curve roughly—an easy matter with even degree curves—to see if the curves are long enough for the desired spiral lengths; 5) if the curves are long enough, compute for each curve θ_s and p: 6) add to the tangent lengths of 2) the quantity p tan $\Delta/2$; 7) recompute the distances between P.I.'s and compute the required shifts in the original tangents as illustrated in Fig. 5 and 6, slide rule precision being sufficient.

If a curve is so short as to make it desirable to replace it completely with spirals, it is necessary to increase the length of spiral because θ_s must be equal to $\Delta/2$ and D_c is also fixed. The required length of spiral is

$$L_s = \frac{200\,\theta_s}{D_c} = \frac{100\,\Delta}{D_c}$$

This length of spiral must be used for both spirals of the short curve and hence for the spirals of the adjacent curves on the side nearer the short curve. With this length of spiral determined steps 5, 6, and 7 may be performed, the only modification being that the curve or curves with unequal

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spirals have two values of p. Calling the smaller and larger offsets p_1 and p_2, respectively, the tangent length of the tangent with the smaller offset is increased by the quantity $p_1 \tan \Delta/2 + \frac{p_2 - p_1}{\sin \Delta}$ and the other by $p_2 \tan \Delta/2 - \frac{p_2 - p_1}{\sin \Delta}$

To illustrate this procedure, the characteristics of the curves and spirals of the four P.I. compound curve used in Example No. 2 will be computed. To emphasize the flexibility of the method, the length of the tangent from P.I. 13 to P.I. 14 will be assumed to be 237.44, the value established in the original solution.

The computations are shown in Table I. The curve designations are shown in Col. 1 with the designation of the forward tangent (P.I. to P.I.) above and that of the backward tangent below. Central angles are shown in Col. 2 and the value of the tangent distance for a 1° curve with that central angle is given in Col. 3. The trial tangent length is given in Col. 4. The value for Curve 11 is assumed to be 265' —fixed by terrain —and for subsequent curves is computed in Col. 7 by subtracting the tangent length of the preceding curve from the distance between P.I.'s. Col. 5 is the quotient of Col. 3 divided by Col. 4. Col. 6 contains the rounded value of the degree of curve and Col. 7 is the tangent length obtained by dividing Col. 3 by Col. 6. Col. 7 also contains the distance between the P.I. of the curve and the next forward P.I. The tangent length of the curve is subtracted from this distance to obtain the trial tangent length of the next curve. The sum of the tangent lengths of adjacent curves is shown in Col. 8 opposite the appropriate P.I. to P.I. designation. Length of circular curve is also shown in this column.

Cols. 1 thru 8 assume an unspiraled compound curve. From Col. 8 it is apparent that 150' spirals will be satisfactory for all curves except Curve 14. To make a 6° curve with a central angle of 10° 10' completely transitional

$$L_s = \frac{100 \times 10.1667}{6} = 169.44$$

Spiral lengths are listed in Col. 9. Curve 13 has unequal spiral lengths so two lines must be used, the one above being for the spiral to Curve 12 and that below to Curve 14.

The spiral angle, θ_s, is listed in Col. 10 and the value of p in Col. 11. The value of p tan $\Delta/2$, listed in Col. 12 must be added to the value of T in Col. 7 to obtain the final tangent length. In addition, when a curve has unequal spirals, the difference in the offsets divided by sin Δ must be added to the tangent length with the shorter spiral and subtracted from that with the longer spiral.

The final P.I. to P.I. distances are listed in Col. 14. These values are used to compute the required tangent shifts as shown in Figs. 5 and 6. Original tangents are shown in solid lines, final tangents in dashed lines. Distances obtained from original and revised tangents lengths are shown without parenthesis. Those values derived from these distances are shown in parenthesis. All values shown in parenthesis are obtained by slide rule.

A summary of the shifts follows:

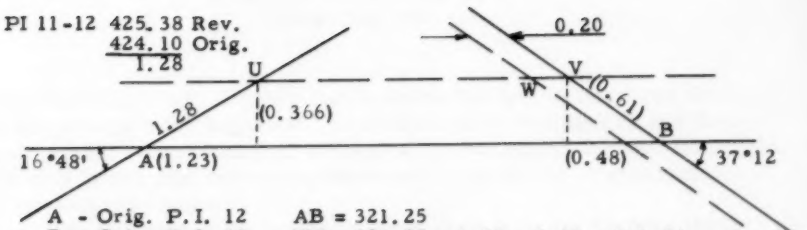
PI 10-11	None
11-12	None
12-13	0.37 left

13-14 0.12 right
14-15 0.98 right

Admittedly, in some cases, adherence to whole degree curves might result in shifts which were too large. In these cases the use of curves to the nearest half-degree should reduce the shifts to reasonable amounts.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
PI-PI Forward / Curve No / PI-PI Backward	Δ	l°Curve T	Trial T	Computed D	Assumed D	Distance PI-PI / Trial T	L_c / Distance PI-PI	L_s	θ_s	p	$p \tan \frac{\Delta}{2}$	Final T	Final Distance PI - PI
11 - 12	° '			°	°	424.10			°				
11	15-16	767.9	265	2.90	3	255.97	509	150	2.25	0.49	0.07	256.04	
10 - 11						168.13							
12 - 13						321.25							
12	16-48	848.1	168	5.03	5	169.22	336	150	3.75	0.82	0.12	169.34	
11 - 12						152.03	425.19						425.38
13 - 14						237.44		150	9.75	2.13	0.72	150.00	
13	37-12	1928.2	152	12.7	13	148.32	286	169.44	11.01	2.71	0.91	148.27	
12 - 13						89.12	317.54			0.58 ÷ sin Δ = 0.96			319.34
14 - 15													
14	10-10	509.7	89	5.72	6	84.95	169	169.44	5.08	1.25	0.11	85.06	
13 - 14							233.27						233.38

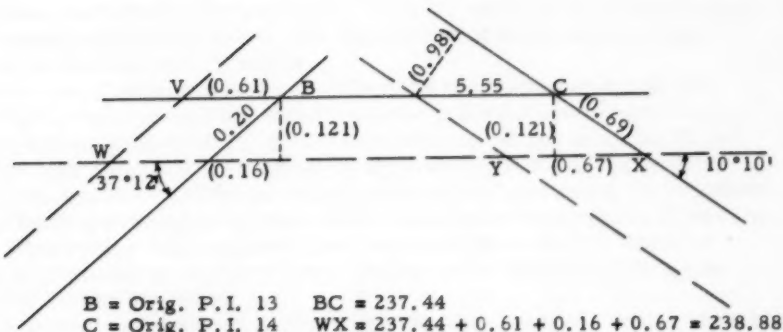
Table I



A - Orig. P.I. 12
B - Orig. P.I. 13
U - Rev. P.I. 12
W - Rev. P.I. 13

AB = 321.25
UV = 321.25 - 1.23 - 0.48 = 319.54
Revised length PI 12-13 = 319.34
W should be moved back = 0.20

Fig. 5



B = Orig. P.I. 13
C = Orig. P.I. 14
W = Rev. P.I. 13
Y = Rev. P.I. 14

BC = 237.44
WX = 237.44 + 0.61 + 0.16 + 0.67 = 238.88
Revised length PI 13-14 = 233.33
Y should be moved back 5.55

Fig. 6

Discussion of
"DETERMINING THE REQUIRED THICKNESS OF CONCRETE PAVEMENTS FOR HIGHWAYS"

by Joseph Herbert Moore
(Proc. Sep. 596)

L. A. PALMER,*—Mr. Moore's paper points the way to improved design of concrete pavement for highways. Two outstanding conclusions are that nothing is gained, economically or otherwise by increasing the slab thickness beyond 8 or 9 inches and that more effort and funds should be spent on improving the subgrade support.

The general tendency of highway engineers is to compute the slab thickness with Pickett's semi-empirical expressions for exterior corner loading, assuming that only about 20 per cent of the load can be transferred from one slab corner to the next. Pickett, of course, assumed in developing his expressions, that for some distance from the corner there is no subgrade support. His expressions are thoroughly reliable for the assumed conditions.

It is both easy and economical to maintain good subgrade support at slab corners and elsewhere. A well compacted subbase of stable earth material and of adequate thickness under a concrete pavement is not a luxury. Where there is irregular and "mushroom" support of the slabs, none of the rational methods of Westergaard, Pickett and others can have any application whatsoever. These methods do hew to the line, however, when there is continuous and permanent subgrade support. The application of sound engineering principles is thereby made possible.

Present specifications and design criteria of the Bureau of Yards and Docks require that the subbase material under concrete pavement have a minimum soaked CBR of 30 and that it be compacted to not less than 95 per cent of the maximum density corresponding to optimum moisture as determined by the modified AASHO laboratory compaction procedure. It is further required that the compacted subbase have a minimum Westergaard K modulus of 200. Where these requirements have been met there is no instance of pumping at joints known to this writer. Neither have there been instances reported of corner cracking.

Where these conditions are realized on highways, thickness design should be based on anticipated maximum bending stresses at points close to the free longitudinal edges as Mr. Moore concludes. At airfields, loads near the free edges are quite infrequent and with good load transfer at transverse contraction and longitudinal keyed joints, design should have reference to loading at the slab interiors.

Where there is plain concrete, the slab length must depend on necessary limitations in openings at contraction joints as well as on restrained warping stresses. These joints must not open so wide as to preclude good aggregate interlock. The quality and maximum size of coarse aggregate is a factor to be considered in this connection.

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Figure 3 of Mr. Moore's paper shows what can be gained by designing relatively thin and heavily reinforced slabs. It shows further that the real advantage of good subgrade support tends to be lost when the thickness of the highway concrete pavement exceeds 8 or 9 inches. Obviously, flexural strength combined with flexibility and also with reduction in restrained warping stresses are in the direction of optimum conditions for longevity of concrete pavement subjected to highway traffic. And it is only another small step from this conclusion to a full appreciation of the remarkable benefits that can be realized by prestressing concrete pavements for highways and airfields. The Department of the Navy has already made a very good preliminary study of the possible advantages to be gained by such prestressing.

An additional point with reference to fatigue effects is that of the time between successive load repetitions. It is obvious, for example, that 50,000 load repetitions at a point with minute intervals between successive loads, is far more damaging than the same number of loadings at hourly intervals. Therefore points near the longitudinal edges are subjected to fewer load repetitions and with greater time intervals between successive loads than other points. The greater this time interval, the more is there of "recovery" from any shock before it is repeated.

EARL C. SUTHERLAND,* M. ASCE.—The subject, discussed by the author, regarding the proper theoretical formula to use for designing the thickness of concrete pavements is one on which there is disagreement at the present time.

At the time that Westergaard made his notable theoretical analysis of the stresses in concrete pavements caused by loads, there were two particular types of structural failure which were of concern. These were:

1) Corner breaks at outside corners and
2) Open transverse cracks which usually spalled badly.

Figure 1 shows an outside corner break of the type analyzed by Westergaard.(1) The crack developed at distances of approximately 36 to 50 inches from the corner. As the author points out there are very few breaks of this type to be found in pavements today. This applies not only to modern pavements, but to older pavements which are still in use. The disappearance of this type of failure is probably largely due to two factors: (1) the substitution of pneumatic for hard rubber tires with a resulting increase in the tire contact area and (2) the increased width of pavements which results in traffic traveling at greater distances from the outside edges of the pavement.

Pavements of modern design are constructed in a manner in which makes transverse cracks less objectionable than formerly. For example, plain concrete pavements are constructed with contraction joints at sufficiently close intervals to control transverse cracking and expansion joints have been largely eliminated so that the contraction joints as well as such cracks as may develop are held closed. Reinforced pavements are constructed with sufficient longitudinal reinforcement to hold transverse cracks closed and this greatly reduces the tendency for spalling or raveling. Also cracks in reinforced pavements have little if any tendency to pump except where the steel has broken.

The slab end failure which has become more common in recent years is perhaps the most troublesome type of failure developing in concrete pavements at the present time. A typical example is shown in figure 2. The

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actual cracking of the slab is usually preceded by severe faulting at the joint. The crack which follows usually is at distances of from 5 to 8 feet from the joint. This type of failure is found only in pavements carrying a considerable number of heavy trucks and is primarily the result of a loss of subgrade support under the slab end. The most common cause of this loss of subgrade support is pumping although consolidation and softening of the subgrade by moisture are also factors.

Unfortunately none of the theoretical formulas for computing stresses in concrete pavements apply for the slab end load condition nor does it seem likely that valid formulas for this load condition can be developed until more experimental data are available on the subgrade support conditions provided by granular subbases on subgrades of a pumping character and on the effectiveness of load transfer in reducing the magnitude of the stresses.

It was stated earlier that transverse cracks in modern concrete pavements do not appear to be as destructive as they were in the pavements of years ago. It is important, however, that pavements be designed with sufficient thickness to prevent the formation of transverse cracks at close intervals. The slabs of plain concrete pavements should be designed of the proper thickness and length to eliminate intermediate cracks as far as possible. Reinforced pavements, of the conventional type containing the usual amounts of longitudinal reinforcement, should be designed with sufficient thickness to avoid transverse cracking at intervals of less than approximately 15 to 20 feet.

There appears to be good justification for the author's recommendation that concrete pavements be designed on the basis of the edge case of loading taking into consideration both load and warping stresses. As a matter of fact the Bureau of Public Roads has for a number of years evaluated the influence of loads on pavements in this way. It is known that if pavements are designed by this procedure slab end failures will not develop, provided that pumping is controlled and effective load transfer is maintained in the joints and cracks.

The Westergaard load stress formula for the edge case of loading(1) was, as the author states, verified in the Arlington investigation.(2) There has, however, never been an adequate experimental investigation to determine the validity of the available formulas for computing warping stresses. The first analysis of warping stresses was made by Westergaard(3) and was limited to slabs of infinite length and finite width. At the time that this analysis was made it was not realized that the temperature differentials that develop in concrete pavements were so large. After seeing some of the temperature differential data obtained in the Arlington investigation, Westergaard expressed the opinion that his analysis might not apply for temperature differentials of these magnitudes.

After it had been demonstrated in the Arlington investigation that the warping stresses in concrete pavements are very important and with no means for computing them for slabs of finite length, Bradbury developed formulas(4) for this purpose based upon the Westergaard formulas for slabs of infinite length.

The measured warping stresses obtained in the Arlington investigation(2) were for one slab length, 20 feet, and for two slab thicknesses, 6 and 9 inches, respectively. The average measured warping stresses, at the free edge, for the summer condition was of the order of 310 pounds per square inch for both the 6- and 9-inch slabs. The corresponding stresses computed by Bradbury's formulas for the 6- and 9-inch slabs are 245 and 315 pounds per square inch, respectively. For this case the computed stress for the 6-inch slab is appreciably smaller than the measured stress. For the 9-inch slab there is good agreement.

Subsequent to the Arlington investigation a considerable number of warping stress measurements were made on a pavement with a 15-foot slab length.(5) The results of this study have not been published.

The pavement on which these tests were made was constructed on the same subgrade as the Arlington pavement after the removal of the earlier pavement. The top 6 inches of the subgrade was stabilized, but this did not make an appreciable change in the modulus of subgrade resistance. The pavement was of 8-inch uniform thickness and the concrete was of two types, one-half contained a gravel coarse aggregate while the other half contained crushed limestone. The physical properties of the two concretes were approximately the same and differed little from those of the concrete in the older Arlington test pavement. The average summertime warping stresses measured on this pavement together with the corresponding computed stresses are tabulated below. The temperature differential used for the computed stresses was essentially the same as the average measured differential.

Average measured and computed warping stresses on pavement slabs of 8-inch uniform thickness and 15 feet in length

Coarse aggregate	Interior		Free edge	
	Measured	Computed	Measured	Computed
	Stress in lbs. per sq. inch		Stress in lbs. per sq. inch	
Gravel	130	240	120	230
Crushed stone	150	265	115	250

Each measured stress value in the above tabulation is the average of approximately 16 readings. It is evident from this tabulation that the measured warping stresses on the 15-foot slabs are much smaller than the computed stresses for both the interior and edge cases of loading. Also the measured warping stresses for the 15-foot slabs at the edge are less than 50 per cent of those measured for the 20-foot slabs in the Arlington investigation. Thus it appears that the warping stresses computed with existing formulas for slabs of finite length are much larger than the actual stresses where the slab length is of the order of 15 feet.

Experience has shown that in many states a 20-foot joint spacing is effective in controlling intermediate transverse cracking while in others, especially the western States, it has been found necessary to reduce the length to 15 feet.

In conclusion 5, the author states that under present day traffic it is not necessary to use slab thicknesses greater than 8 or 9 inches. This, of course, applies only where subgrade support is maintained. The conclusion is supported by data obtained in experimental investigations as well as the theoretical studies discussed by the author. For example, on Road Test 1 - MD. the maximum stresses measured for the 22,400-pound single axle and the 44,800-pound tandem axle were of the order of 200 pounds per square inch or less on the natural granular subgrade on which no pumping developed.

In conclusion 6, it is stated that funds should be spent in improving the bearing capacity of the subgrade rather than for increased thickness of pavement. This is true where the subgrade is fine grained material and where there is danger that pumping may develop. Where there is no danger of

Figure 1.—Corner break typical of those which were common in pavements years ago.

Figure 2.—Slab end failure caused by pumping.

pumping, however, it is likely to prove more economical to increase the thickness of the pavement slab rather than to attempt to increase the bearing power of the subgrade since there is considerable evidence that granular subbases of usual thicknesses do little to increase the supporting power of the subgrade.

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WARNER HARWOOD,* A.M. ASCE.—Mr. Moore's paper focuses attention on the effect of increased width of pavement lanes in decreasing the proportion of applications of load which occur at critical locations close to the edge of the slab. However, by certain assumptions used in the determination of the load and warping stresses, he arrives at values for these stresses which are subject to question.

This paper is largely a summary of the material on rigid pavements originally presented in the "Technical Supplement to Highway Use and Highway Costs, A Report of the Joint State Government Commission to the General Assembly of the Commonwealth of Pennsylvania, Session of 1953." Table 2 of Mr. Moore's paper is a copy of Table A-6 of that report, which is based on the computations of load and warping stresses shown in detail in Table A-5.

A study of these tables indicates that for all loadings the edge warping stress in the 8" slab is computed as 251 psi for a temperature differential of 3 degrees F. per inch of depth and 167 psi for a 2 degree F. differential. These were computed by the formula proposed by R. D. Bradbury. Stresses as computed by these formulas seem to be unreasonably high. When they were published by the American Concrete Institute in 1939 in a paper "Application of the Results of Research to the Structural Design of Concrete Pavements" by E. F. Kelley, Chief, Division of Tests, Bureau of Public Roads, a discussion was submitted by A. T. Goldbeck, Engineering Director, National Crushed Stone Association. Mr. Goldbeck was Mr. Kelley's predecessor as Chief, Division of Tests, of the Bureau of Public Roads. In this discussion Mr. Goldbeck stated:

† Note: Reference 9 in the author's paper should be corrected to correspond with this reference.

* Highways and Municipal Bureau, Portland Cement Ass'n., Chicago, Ill.

"I think that perhaps one of the phases of Mr. Kelley's paper which will be surprising to many, deals with the question of warping stresses. Warping stresses are really bending stresses and are shown to be very high, particularly when the slabs have lengths to which we are accustomed, for instance, 30 feet. . . . . I am not convinced that the results obtained from the formulas used for calculating warping stresses are entirely correct. In this connection, let me refer to Table 9 in which are shown transverse temperature warping stresses. The effect of transverse warping is to produce bending of the slab transversely. It would seem that the greatest transverse stress which could possibly exist would be that in which the slab is so warped that it rests upon a line of support at the center of the slab. The dead load stress for a ten-foot width of a 6-in. slab thus supported is 156 psi., whereas the warping stress for a modulus of subgrade reaction of 300, is given at 210 psi. For an 8-in. slab the maximum possible dead load stress at the center, assuming the slab supported at the center and overhanging at the end, is only 117 psi. as against a calculated warping stress of 190 psi. This does not seem logical to me and I am wondering if the warping stresses in a longitudinal direction are really as high as the calculations show. . . . . I note that warping stresses were observed on a test slab at Arlington. It would be very interesting to me to know how such stresses were observed. It is obvious that if deformations are taken with a strain gage, such deformations might give misleading results. It is perfectly possible, for illustration, to have high stress where no deformation is observable whatever; and, correspondingly have zero stress in spite of what seems to be a deformation."

Little additional data had been published on warping stresses until the "Final Report on Road Test One-MD" included Fig. 151 which Mr. Moore has used as Fig. 1 of his paper. Here the stresses are of considerably greater magnitude than those which would be found by the Bradbury formula. This discrepancy casts further doubt upon the accuracy of both the method of computing the stresses and the determination of these stresses in the field by the use of strain gages. In speaking of this figure, the Maryland report says:

"The data of Fig. 151 indicate that the daytime warping stresses are quite high at some distance from the ends of the pavement slabs. Stresses of such magnitude when combined with the daytime stresses of the interior and free-edge loadings approach the modulus of rupture of the concrete, and as a result, transverse cracking might be expected. No significant cracking of the type attributable to the combined stress condition of these cases of loading was observed in this investigation."

The pavement used in the Maryland Test Road had been in service for 9 years before the test started. It should be noted that at the time the test started there was little transverse cracking on the project and none in any of the slabs which were on a granular subgrade. The only transverse cracks which formed during the test loading were in slabs on fine grained soils where subgrade failure occurred during the test. The calculated warping stresses added to the observed load stresses certainly exceeded the modulus of rupture of the concrete. Since no cracks formed it is evident that the actual stress was less than the sum of the calculated and observed stresses. Probably both the "calculated" and "observed" stresses were greater than the actual stress.

In Mr. Moore's computation of the stresses due to load there is also an assumption which results in higher values than exist in the field. For a wheel load placed so that the center of the wheel (or outside tire of dual-tired wheels) is 6 inches from the edge of the slab, he computes the stress by the Westergaard formula for edge stress. This formula considers the contact area as a semicircle whose diameter is along the pavement edge, and its use results in higher edge stresses than are present under the loading in question.

For example, for a 9,000 lb. wheel load the center of gravity of the semicircular contact area as used in the Westergaard formula is 4.1 inches from the edge of the slab. In contrast the center of gravity of a dual-tired wheel would be 12.5 inches from the edge if the center of the outside tire is 6 inches from the edge. This increase of over 8 inches would decrease the stress about 30 per cent according to Road Test One-MD data and approximately 25 per cent when computed by the use of influence charts as outlined in Pickett and Ray's ASCE paper (Transactions, Vol. 116) "Influence Charts for Concrete Pavements." A proportionate reduction should be applied to the load stresses for other locations as they are all based on the stress computed for the load 6 inches from the edge.

The result of these improper assumptions is most forceably brought out in Fig. 3 of Mr. Moore's paper. This indicates that a pavement 7" in thickness on a subgrade with a "k" value of 300 psi/in. would have about 20 times the load carrying capacity of a 12" pavement on the same subgrade. While Mr. Moore points out in Table 9 the absurdity of the long lives shown for corner loading, the extremely short lives shown in the same table based on longitudinal edge loading are equally absurd. First, both the warping and the load stresses which are assumed to be combined to produce the longitudinal edge failure are too high as pointed out earlier in this discussion. Second, these lives are arbitrarily taken as twice the age at which a transverse crack would form. Experience gives no justification for this assumption. Plain pavements are jointed to prevent transverse cracking. Reinforced pavements contain steel to prevent intermediate transverse cracks from becoming points of weakness. In neither case is there any reason to establish pavement life on the basis of age at which the first crack forms.

ALBERT de NEUFVILLE.[1]—The author's statement that concrete "pavements of 8 or 9 inch thickness give longest life" (Synopsis, second paragraph and a similar statement under Conclusions no. 5) seems to be questionable. This conclusion is apparently based on the computation of combined load and restrained warping stresses along the longitudinal edge, as shown in author's Fig. 2. The computation of the restrained warping stresses, however, is based on Westergaard's assumption of a constant temperature gradient in the pavement, an assumption which is inconsistent with the laws of heat flow.

Unfortunately, the author in his search of the literature seems not to have run across a paper by Thomlinson[2] in which he developed the necessary formulas for temperature stresses in concrete pavements where the temperature distribution is consistent with the laws of heat flow. The variations of temperature in a pavement during a daily cycle as shown in Fig. 2 of

1. Asst. Prof. of Mechanics, Lehigh Univ., Bethlehem, Pa.
2. "Temperature Variations and Consequent Stresses Produced by Daily and Seasonal Temperature Cycles in Concrete Slabs" by J. Thomlinson, Ph.D., B.Sc. Concrete and Construction Engineering, vol. 35, June and July 1940, pp. 298 and 360.

Thomlinson's paper corresponds fairly well with the temperature measurements in the Arlington Tests.[3] Thomlinson's Table I shows that the internal stress plus the restrained warping stress is always smaller at the bottom of the slab than at the top. For a 6-in. pavement the bottom stresses are only about 70 per cent of the top stresses, while for a 12-in. pavement this percentage decreases to about 50 per cent. The numerical values of the stresses computed according to Thomlinson's method cannot be directly compared with the stresses found with Bradbury's formula: in it the temperature difference between the top and the bottom of the slab has to be assumed, whereas Thomlinson's analysis is based on the temperature variations at the top of the slab which are independent of the thickness of the pavement, but rather are determined by climatic conditions. Under Thomlinson's assumptions the highest restrained warping stress of about 280 psi at the bottom of the slab occurs in pavements of 8 to 9 inch thickness, while thinner or thicker pavements are subjected to lower stresses at this point under these conditions.

The author's reference to the results of Road Test One-Md (his Fig. 1) is not convincing because the Maryland tests which are concerned with restrained warping stresses apparently were evaluated with the probably outdated assumptions of Westergaard.

Lack of time and space prohibit the writer's presenting the case for thicker concrete pavements in this discussion. He would like to express the opinion, in brief, that a correct evaluation of warping stresses may, from a structural as well as from an economical point of view, justify the place concrete pavements thicker than 8 to 9 inches have in our road system.

3. Public Roads, vol. 16, No. 9, Nov. 1935, p. 177, Fig. 9

Discussion of
"SIGHT DISTANCE AT UNDERCROSSINGS"

by Paul Hartman
(Proc. Sep. 597)

T. F. HICKERSON,[1] M. ASCE.--As a partial check on Professor Hartman's formulas applying to Case II, let d = 0, then k = 1/2; hence F, the quantity in the bracket of Eq. 10, becomes

$$F = \frac{S}{4} - \frac{C - \frac{1}{2}(h+h_1)}{g_2-g_1} \qquad (24)$$

With C = 14', h = 6', and h_1 = 1.5',

$$F = \frac{S}{4} - \frac{10.25}{g_2-g_1} \qquad (25)$$

Substituting Eq. 25 in Eq. 11, one gets

$$L = 8F = \frac{S}{2} - \frac{82}{g_2-g_1}, \qquad (26)$$

which agrees with Eq. 17, the AASHO formula.

In view of Professor Hartman's conclusions and comments, the writer is more convinced than ever that the conventional formulas are good enough if attention is called to the range of errors (on the side of safety) for the rather extreme situations (in Case II, or IIa) where the critical edge is more than 200 feet from the vertex of the vertical curve. The conventional formulas possess the great advantage of simplicity in derivation and application. Since the hazards of passing sight distances are involved in most Case II practical problems, the erroneous results are rendered less important.

As a concluding comment, the writer considers Professor Hartman's paper to be a very able and valuable contribution.

1. Prof. of Applied Math., Univ. of North Carolina, Chapel Hill, N. C.

Discussion of
"PROCEEDINGS - SEPARATES 276, 320, 351, 357, 500"

(Proc. Sep. 598)

CORRECTIONS TO 357:

Correction to Discussion of "A Mathematical Examination of Spiraled Compound Curves" by T. F. Hickerson (Proceedings Separate No. 357).

Page 598-16. The second line under the table should read:

$$\frac{160}{200} \times 4^{o} + (\frac{160}{600})^{2} \times 120' = 3^{o}\,20.5' .$$

PROCEEDINGS-SEPARATES

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

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JUNE: 444(SM)[e], 445(SM)[e], 446(ST)[e], 447(ST)[e], 448(ST)[e], 449(ST)[e], 450(ST)[e], 451(ST)[e], 452(SA)[e], 453(SA)[e], 454(SA)[e], 455(SA)[e], 456(SM)[e].

JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)[c], 488(ST)[c], 489(HY), 490(HY), 491(HY)[c], 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)[c], 502(WW), 503(WW), 504(WW)[c], 505(CO), 506(CO)[c], 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

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NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

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FEBRUARY: 608(WW), 609(WW), 610(WW), 611(WW), 612(WW), 613(WW), 614(WW), 615(WW), 616(WW), 617(IR), 618(IR), 619(IR), 620(IR), 621(IR)[c], 622(IR), 623(IR), 624(HY)[c], 625(HY), 626(HY), 627(HY), 628(HY), 629(HY), 630(HY), 631(HY), 632(CO), 633(CO).

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APRIL: 659(ST), 660(ST), 661(ST)[c], 662(ST), 663(ST), 664(ST)[c], 665(HY)[c], 666(HY), 667(HY), 668(HY), 669(HY), 670(EM), 671(EM), 672(EM), 673(EM), 674(EM), 675(EM), 676(EM), 677(EM), 678(HY).

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c. Discussion of several papers, grouped by Divisions.

e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

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